Geoff Lehman and Michael Weinman (2018). *The Parthenon and Liberal Education*. Albany, NY: State University Press. Pp. xxxiii+234. ISBN:978-1-4384-6841-9; $90.00

Although it is generally not advisable to look for invisible things, it is true that quite visible and worthwhile items do turn up along the way. It is also true that what is admittedly invisible may have only *become* so, either by being lost—like missing tesserae of a mosaic or a lacuna in a text or a missing text in a corpus of texts—or, more significantly, what has vanished may have fallen away or been thrown away, having served its purpose, like the scaffolding of a building. This may happen on a conceptual level too, that is, where what is vanished is not so much a *something* as a role or meaning of something that itself has not vanished. In the history of ideas, that role may be as a mediator between ideas from one time and those from another. To this end, the political theorist Fredric Jameson invented the term "vanishing mediator," which he used to describe Weber's narrative, among others, of how Protestantism mediated the development of capitalism out of feudalism. Jameson says that such a "vanishing mediator," "...serves as a bearer of change and social transformation, only to be forgotten once that change has ratified the reality of the institutions."[1]

In their book, *The Parthenon and Liberal Education,* Geoff Lehman and Michael Weinman, adopting this line of thought, want to show that the very visible "...Parthenon is the most important 'vanishing mediator' between the archaic and largely 'illegible' reception of Near-Eastern knowledge practices in Greece during the seventh and sixth centuries BCE and the creation of Plato's Academy" (p.1). In fact, Near-Eastern knowledge, as such, plays little part in their narrative. The best we get on this front are vague references, such as this in the introduction:

> Noticing the relevance of a problem-based, "trial and error" approach to theoretical mathematics that links the work of Philolaus to mathematical practice in the seventh- and sixth- century BCE Near-Eastern astral science provides insight into how mathematical knowledge procedures functioned in the Parthenon. We see in the temple's design [i.e. the Parthenon's design] precisely the kind of "algorithmic problem-oriented style" we know to be the hallmark of Near-Eastern approaches. (pp.xxx-xxxi)

To sum up Near-Eastern knowledge as "algorithmic and problem-oriented" and to make that the source of both mathematical and architectural practice, in my view, says both too little and too much. But we can leave that aside, for, when it comes down to it, what Lehman and Weinman really want to join by way of the Parthenon is, on one end, an earlier mathematical, especially musical, thinking such as that represented by Philolaus and, on the other end, Plato's view of how arithmetic, geometry, astronomy,

[1] Jameson, F. (1973). "The Vanishing Mediator: Narrative Structure in Max Weber." *New German Critique,1*, 52–-89 (p.80). In the sentence quoted, Jameson is referring specifically to the idea of institutionalized charisma.

and music—the later *quadrivium*—form essential stepping stones to dialectic, that is, how the mathematical arts, in this sense, function as true *liberal arts*.

That music is central to the thought of Philolaus and the Pythagoreans is incontrovertible. It is also central to the proportions in the design of the Parthenon. And, as we learn from this book, music is central in at least certain key arguments in Plato. The thorough and rich discussions of these points taken individually, particularly the design principles of the Parthenon discussed in part II of the book (Lehman) and the insights into Plato's *Timaeus* and the *Republic* in part I (Weinman) are among the genuinely worthwhile things found along the way as Lehman and Weinman search for a nexus between mathematical investigations into music, the Parthenon, and liberal education.

For example, as an interpretation of Plato, the discussion of *Republic*, IX.587b–588a is quite enlightening. This is the strange passage in which Socrates tries to persuade Glaucon that the life of a wise king is measurably more pleasant than that of the tyrant, concluding that the former is 729 times more pleasant, no more and no less. Though the argument is dubious, the number 729 is not arbitrary. Plato alludes to the arithmetical fact that 729 is both a square and a cube ($729=9^3=27^2$), a plane and a solid. And he says that "number is true…and appropriate to lives (*alēthē kai prosēkonta ge…biois arithmon*), if days and nights and months and years are appropriate to them" (588a), almost certainly referring to Philolaus' Great Year in which the solar year and lunar month coincide, namely, a period of 729 lunar months, or, alternatively, to the 729 days and nights in a single Philolauian year (=364 ½ days).

But the number 729 also appears in a musical context, as Plato well knows, and Weinman abundantly points out. The number 729 occurs in the problem of dividing the octave. *Timaeus* 35c–36a is based on the Pythagorean approach to the problem and may be associated with Philolaus. Accordingly, the octave, the musical interval corresponding to the ratio 1:2, is viewed in terms of the basic perfect intervals, the fifth, 3:2, and the fourth, 4:3, and their difference, the tone, 9:8 (i.e. 3:2÷4:3). Philolaus knew that the perfect fourth and the perfect fifth add up to the octave (3:2×4:3=1:2). Three whole tones produce an interval greater than the fourth and less than the fifth and thus comes close to dividing the octave. The numerical ratio corresponding to three whole tones is, precisely, 729:512 (9:8 compounded twice, or in modern terms 9/8×9/8×9/8), whence 729.

Now, this interval, 729:512 differs from the perfect fifth by the ratio of 256:243 (3/2 ÷729/512). The *Timaeus* passage stops at this "part left over," but the problem beneath the surface is that 729:512 differs from the perfect *fourth*, 4:3, by a ratio of 2187:2048 (729/512÷4/3), and this is *not* the same ratio as 256:243. Since, on the other hand, perfect fourth and the perfect fifth add up to the octave (3:2×4:3=1:2), 729:512 does not *exactly* divide the octave. The difference between the two left-over intervals, 256:243 and 2187:2048, became known in later music theory as the Pythagorean comma, the Pythagorean gap. Indeed, it is *not possible* to divide the octave with such whole number ratios, as was shown by Plato's friend, Archytas (presumably, along the lines of Euclid's *Sectio Canonis*, prop.3). A gap is unavoidable.

In view of the *Timaeus* passage and Plato's friendship with Archytas, one would expect Plato to refer to music in the passage from the *Republic*, as he does arithmetic, astronomy, and, perhaps, geometry. Like the opening of *Timaeus*, "One, two, three—but where is the fourth?," Weinman surmises that the absence of music here is meant to call attention to music. And the gap is all the more gaping when we realize what should be said about music has itself to do with a gap, and a kind of incompleteness. Plato, in Weinman's view, wants to draw our attention to the fundamental incompleteness of the Pythagorean way of understanding the world and, by implication, of the mathematical way of understanding the world. It is this realization of the limits of mathematics that make its study a prelude to dialectic:

> ...in our view Plato does not identify directly with the Pythagorean project of finding the highest truth (the good) through the application of the mathematical arts in understanding the cosmos, but rather places this approach in a subordinate role to the dialectical approach to the (form of the) good. Plato aims to place these insights as a serious but ultimately hopelessly incomplete attempt to answer, with mathematics, questions that can only be answered dialectically. This, we suggest, is the relationship of mathematics to philosophy and the meaning of the suggestion that the arts are a "prelude" to the song of dialectic (*Resp*. 7.531d)...(p.37)

Thus Plato.

Like music, or even directly inspired by music, fifth century architecture too is based on whole number ratios—indeed, like the musical scale, on *systems* of proportions of whole number ratios. This is particularly so for the Doric order of architecture, of which the Parthenon is a prime example. As Lehman tells us, the Doric order,

> ...is in many ways the development, in concrete and visible terms, of a system of *symmetria*. Doric is characterized by relationships between repeating sequences of similar elements at different scales...that lend Doric buildings a part of their "organic" quality: like the members of a body, the smaller elements relate to the larger ones both numerically and formally, all contributing to the being and to the beauty of the whole. (pp.72–73)

Lehman presents the complexity of these systems of proportions in welcome detail in part II of the book (especially chapter 4). The systems are in some ways even more complex than those in music: the problem of making a building fit together on the basis of simple whole number ratios—to harmonize (recalling that *harmozein* means, literally, to fit together)—is at least as thorny as producing a Pythagorean scale and similarly produces gaps and incompleteness in the what should be an "organic" whole. For example, we have the "corner problem" in which the proportions of triglyph and metope in the entablature must be fitted together with the intercolumination and the stylobate[2] so that everything come out evenly at the corners: the difficulty is that one part of the system uses 9:4 as its basic unit (in musical terms two fifths: 3:2×3:2), that is, 81:16, and another the ratio 5:1, that is, 80:16. So the ancient architects were confronted with a gap, the discrepancy between 81:16 and 80:16, analogous to the Pythagorean comma.

There are important mathematical ideas connected to these problems, both those from music and those from architecture. The problem of combining fifths into octaves, ratios compounded from the ratio 3:2 into those compounded from 2:1, is the problem of *symmetria*, common measure, and of the uniqueness of representing numbers as products of primes, proven in Book VII Euclid's *Elements* (*Elem*.VII.30 also *Elem*.IX.14). The ultimate solution to the problem, the actually fitting together of the elements, the problem of *harmonia*—and Lehman and Weinman are right to separate *harmonia* from *symmetria*—is connected to the incommensurables.[3] Dividing the octave, 1:2, comes down to finding a geometric mean between 1 and 2. Taken as two lengths, where one is twice the other, the mean turns out to be incommensurable with both. The problem of incommensurables, along with the problems of music, was certainly on the minds of Plato and his circle. This is evident not only from what has already been said but also from dialogues such as the *Menon* and *Theaetetus*.

But while incommensurables and the deeper problems connected to music theory may be inherent in the design of Parthenon, that is, visible to one *already* familiar with them, this does not mean that the Parthenon was *intended* to be the bearer of those mathematical difficulties and, therefore, that it mediated the development of Plato's thought regarding educational value of mathematics as the prelude to dialectic. Yet, this is what is claimed: the design of the Parthenon "engages with what was probably the most innovative mathematics of the mid-fifth century" (p.105) and, as with Plato, it does so in a way that brings the viewer beyond it (see p.104). The authors are cognizant of the difficulties in their thesis and to their credit try to answer possible objections of "less-given-to-speculation colleagues" (p.xix). I am afraid, though, I consider myself among the latter and remain not entirely persuaded.

My own difficulties fall broadly on two fronts. The first has to do with the content and character of mid-fifth century mathematics. The second has to do with whether engaging with this kind of mathematics can plausibly be taken as the intention of Iktinos and Kallicrates, the supposed architects of the Parthenon. Of course, the second depends on the first.

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[2] The triglyphs are blocks along the frieze of the entablature marked by three vertical bars. The spaces, often decorated with reliefs, between the triglyphs are the metopes. The stylobate is the base of the columns.

[3] Two magnitudes are incommensurable if they cannot be related by a ratio of a whole number to another whole number. For example, the diagonal of a square is incommensurable with its side.

Regarding the second, it is difficult to know the architects' intentions without explicit texts. Lehman and Weinman do recall a text by Iktinos and Karpion [a possible third architect of the temple] on the Parthenon along with other texts on the Doric order mentioned by Vitruvius (*De Arch*. VII, Intro.12) (pp. xix,150); however, they say that the existence of these texts are not necessary for the main argument. They take the building itself is a kind of text, and their reading of it, as presented in part II of the volume, may be "akin to what this treatise [by Iktinos and Karpion] would have presented," and thus, they believe, will show that "the designers of the Parthenon saw their creation in this way, and that they did so because of their vision of what we might call a liberal education" (p.xix).

Besides the circularity of the argument here, I have difficulty finding a similar view to theirs in one text we do have, that of Vitruvius himself. For example, although Vitruvius speaks about the corner problem, mentioned above, he does not conclude that it was embraced as a means to educate the viewer; rather, he says only that the problem may have been the reason the ancients avoided the Doric order altogether in their temples (*De Arch*. IV, 2.2)! This is striking not the least because Vitruvius speaks about liberal education at the beginning of the work and demonstrates considerable understanding of music theory (esp. *De Arch*.V, 4 and 5).

Even if it were not the intention of Iktinos and Kallicrates to engage with the kind of mathematical problems that interested Plato, it might be argued that the temple was nevertheless a focus for those who *were* thinking about problems of music theory and the theory of incommensurables. But this presupposes that those problems were in the air and the subject of intense work. For this reason, the state of mathematics in the mid-fifth century, my first difficulty, is absolutely *crucial* for the authors' thesis.

The question of when and how the study of incommensurables began is still not completely settled. The authors discuss the positions of Szabó and Knorr as extremes on this question. Knorr claimed that the idea of incommensurables arose only at the end of the 5th century, and that a fully developed theory of incommensurables belonged to the time of Plato. Szabó, who was a philologist, recognized in the later language of ratio and proportion origins in the theory of music, in particular, in its use of intervals as real lengths, such as those of strings, to speak about numerical ratios. Largely on that basis, Szabó placed the origin of the theory of ratio much earlier in the 5th century, as early as the building of the Parthenon.

The authors claim to take a middle position by adopting the view (also to a certain extent found in Szabó) of David Fowler who wrote extensively on the possibility that *anthyphairesis*, "continuous reciprocal subtraction" akin to the Euclidean algorithm, was the true beginning of theory of ratio. *Anthyphairesis* does play a part in Euclid's Book X; however, as a theory of ratio, there is much circumstantial evidence but hardly any textual evidence—chiefly, a single sentence in Aristotle. In any case, the authors' treatment of it as a "practical method" is not what Fowler had in mind. It was supposed to be, in Fowler's view, the center of *theoretical* investigations; it was not a case of "mathematical *practice* [being] well 'ahead' of its theorization and formalization" (p.xxv). It may well be that music and even a primitive form of anthyphairesis were known earlier than the fully developed theory, but one must be very careful about assuming that that theory was, nevertheless, in place and known implicitly. This truly can become looking for the invisible.

My own view is that the weight of evidence favors Knorr; I also tend to agree with his view vis-a-vis music and incommensurability that "...the harmonic theory of irrationality was a derivative from the geometric theory, rather than the converse."[4] A full elaboration of the Knorr's arguments as well as the deficiencies I find in Fowler and Szabó would, however, require a much longer review. Whatever my own conclusions, though, suffice it to say that the position that already in the mid-5th century there existed a genuine awareness of issues connected to incommensurability and even, perhaps, the kind of musical knowledge represented by Euclid's *Sectio Canonis,* does not enjoy the complete consensus among historians necessary to make it a safe foundation on which to base such a bold thesis as this book wishes to make.

In sum, while I have no doubt that the Parthenon could serve as an object for reflection for those educated *after* Plato, I remain unconvinced that it served that role *before*. Thus I cannot be persuaded of its identity as a "vanishing mediator" leading to Plato's idea of the true liberal arts: so adding to the list given at the outset, this role may be invisible because it is not really there. But though I am doubtful of the main thesis of Lehman and Weinman's book, I can still say that I learned much from it, as I hope this review has conveyed. It is a case in which the trees are simply more worthwhile than the forest.

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[4] Knorr, W. R. (1975). *The Evolution of the Euclidean Elements*. Dordrecht: D. Reidel Publishing Company, p.216.